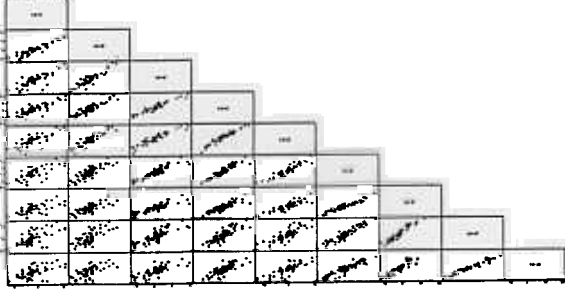


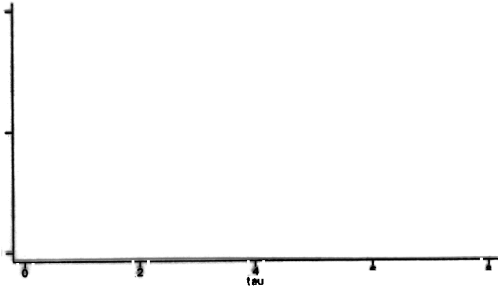
48 pigs

9 week

Autocorrelation Scatterplot



ACF



autocor weekr time Id

	time1	time2	time3	time4	time5	time6	time7
time1	1.0000						
time2	0.9156	1.0000					
time3	0.8015	0.9118	1.0000				
time4	0.7958	0.9084	0.9582	1.0000			
time5	0.7494	0.8809	0.9280	0.9621	1.0000		
time6	0.7051	0.8353	0.9058	0.9327	0.9219	1.0000	
time7	0.6551	0.7759	0.8435	0.8681	0.8546	0.9633	1.0000
time8	0.6255	0.7133	0.8167	0.8293	0.8104	0.9280	0.9586
time9	0.5581	0.6638	0.7689	0.7856	0.7856	0.8893	0.9170
	time8	time9					
time8	1.0000						
time9	0.9695	1.0000					

	acf
1	.9425781
2	.8870165
3	.8462396
4	.7962576
5	.7724156
6	.7121489
7	.6407955
8	.5581002

ANOVA TABLE

Y_i groups to compare

$$Y_i = (Y_{i1}, \dots, Y_{im})'$$

variation in all observations = var. between each obs. and its group mean

+ var. between each group mean and the overall mean

$$SST = SSW + SSB$$

ANOVA TABLE

Source	SS	DF	
Model (between)	$\sum_i \sum_j (Y_{ij} - \bar{Y})^2$	$n-1$	$F_b^2 = \frac{SSB}{n-1}$
Residual (within)	$\sum_i \sum_j (Y_{ij} - \bar{Y}_{i.})^2$	$N-n$	$F_w^2 = \frac{SSW}{N-n}$
Total	$\sum_i \sum_j (Y_{ij} - \bar{Y})^2$	$N-1$	$F_t^2 = \frac{SST}{N-1}$

$$N = n \cdot m \text{ (total \# of obs.)}$$

xtsumcorr weekr

Variable		Mean	Std. Dev	Min	Max	Observations
weekr	overall	3.04e-09	4.386947	-11.90509	13.17512	N = 432
	between		3.953498	-7.405093	9.372685	n = 48
	within		1.976177	-7.728472	8.160417	T = 9
	corr. between		3.855132			
	corr. within		2.093625			
	rho		.7722			

$$\sigma_w^2 = \sqrt{4.38} = 2.09$$

$$\sigma_b^2 = \sqrt{19.25 - 4.38} = 3.86$$

anova weekr Id

Number of obs = 432 R-squared = 0.7971
Root MSE = 2.09362 Adj R-squared = 0.7722

Source	Partial SS	df	MS	F	Prob > F
Model	6611.55323	47	140.671345	32.09	0.0000
Id	6611.55323	47	140.671345	32.09	0.0000
Residual	1683.17352	384	4.38326437		
Total	8294.72675	431	19.2453057		

$$\rho = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_w^2}$$

$$\rho = \frac{\sigma_b^2}{\sigma^2}$$

$$\sigma_w^2 = \sigma_e^2$$

within-subject correlation

OLS

$$Y_{ij} = \beta_0 + \beta_1 t_j + \varepsilon_{ij}$$

$\varepsilon_{ij} \sim N(0, \sigma^2)$

$\sigma^2 = 1.9$

$\sigma = 1.4$

regress week time

Source	SS	df	MS	Number of obs =	432
Model	111060.882	1	111060.882	F(1, 430) =	5757.41
Residual	8294.72677	430	19.2900622	Prob > F =	0.0000
Total	119355.609	431	276.927167	R-squared =	0.9305
				Adj R-squared =	0.9303
				Root MSE =	4.392

week	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
time	6.209896	.0818409	75.88	0.000	6.049038 6.370754
_cons	19.35561	.4605447	42.03	0.000	18.45041 20.26081

$$E[Y_{ij}] = \beta_0 + \beta_1 t_j$$

$$V(Y_{ij}) = \sigma^2$$

$$\text{Cov}(Y_{ij}, Y_{ik}) = 0$$

$i = 1, \dots, m$
 $j = 1, \dots, n$

Uniform correlation model

$$Y_{ij} = \beta_0 + \beta_1 t_j + \varepsilon_{ij}$$

$$E(Y_{ij}) = \beta_0 + \beta_1 t_j$$

$$V(Y_{ij}) = \sigma^2$$

$$\text{Cov}(Y_{ij}, Y_{ik}) = \rho$$

xtgee week time, i(Id) corr(exc)
 Iteration 1: tolerance = 5.585e-15

GEE population-averaged model

Group variable:

Link:

Family:

Correlation:

Id
 identity
 Gaussian
 exchangeable

Number of obs = 432
 Number of groups = 48
 Obs per group: min = 9
 avg = 9.0
 max = 9
 Wald chi2(1) = 25337.48
 Prob > chi2 = 0.0000

Scale parameter:

19.20076

week	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
time	6.209896	.0390124	159.18	0.000	6.133433	6.286359
_cons	19.35561	.5974055	32.40	0.000	18.18472	20.52651

xtcorr

Estimated within-Id correlation matrix R:

	c1	c2	c3	c4	c5	c6	c7	c8	c9
r1	1.0000								
r2	0.7717	1.0000							
r3	0.7717	0.7717	1.0000						
r4	0.7717	0.7717	0.7717	1.0000					
r5	0.7717	0.7717	0.7717	0.7717	1.0000				
r6	0.7717	0.7717	0.7717	0.7717	0.7717	1.0000			
r7	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	1.0000		
r8	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	1.0000	
r9	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	0.7717	1.0000

$$V = \sigma^2 [(1-\rho)I + \rho J]$$

$$V_0 = \begin{pmatrix} 1 & \dots & \rho \\ \vdots & & \vdots \\ \rho & \dots & 1 \end{pmatrix}$$

$$V = \sigma^2 \begin{pmatrix} 1 & \dots & \rho \\ \vdots & & \vdots \\ \rho & \dots & 1 \end{pmatrix}$$

$$Y_{ij} = \beta_0 + U_{ji} + (\beta_1 + U_{2ij}) x_j + \varepsilon_{ij}$$

Random effect model w/ random intercept

$$Y_{ij} = \beta_0 + U_i + \beta_1 x_j + \varepsilon_{ij}$$

$$U_i \sim N(0, \sigma_u^2)$$

$$\varepsilon_{ij} \sim N(0, \sigma_w^2)$$

$$E[Y_{ij} | U_i] = \beta_0 + \beta_1 x_j$$

$$V[Y_{ij} | U_i] = \sigma^2 = \sigma_u^2 + \sigma_w^2$$

. xtreg week time, re i(Id)

Random-effects GLS regression
Group variable (i): Id

Number of obs = 432
Number of groups = 48

R-sq: within = 0.9851
between = 0.0000
overall = 0.9305

Obs per group: min = 9
avg = 9.0
max = 9

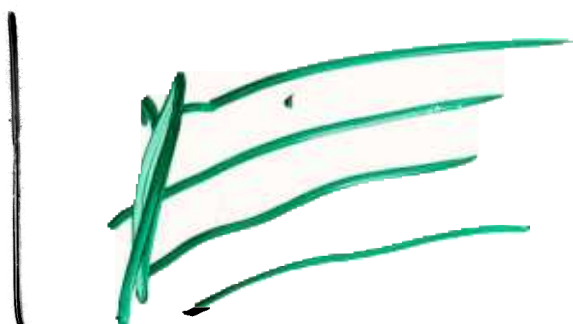
Random effects u_i ~ Gaussian
corr(u_i, X) = 0 (assumed)

Wald chi2(1) = 25271.50
Prob > chi2 = 0.0000

week	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
time	6.209896	.0390633	158.97	0.000	6.133333	6.286458
_cons	19.35561	.603139	32.09	0.000	18.17348	20.53774
sigma_u	3.8912528					
sigma_e	2.0963561					
rho	.77505203	(fraction of variance due to u_i)				



Random
Slope



Random intercept



Both

Exponential correlation model

$$Y_{ij} = \beta_0 + \beta_1 t_j + \epsilon_{ij}$$

$$E(Y_{ij}) = \beta_0 + \beta_1 t_j \quad E(Y_{ij} + Y_{ik}) = 2\beta_0 + \beta_1(t_j + t_k)$$

$$V_0 = \begin{pmatrix} 1 & \rho & \dots & \rho^8 \\ \vdots & \vdots & \ddots & \vdots \\ \rho^8 & \dots & \dots & 1 \end{pmatrix}$$

$$\text{Cov}(Y_{ij}, Y_{ik}) = \sigma^2 \rho^{|j-k|}$$

xtgee week(time, i(Id) corr(ar1)

Iteration 1: tolerance = .02513276
Iteration 2: tolerance = .00009237
Iteration 3: tolerance = 4.366e-07

GEE population-averaged model

Group and time vars:

Link: identity
Family: Gaussian
Correlation: AR(1)

Number of obs = 432
Number of groups = 48
Obs per group: min = 9
avg = 9.0
max = 9
Wald chi2(1) = 6254.91
Prob > chi2 = 0.0000

Scale parameter:

19.26754

week	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
time	6.272089	.0793052	79.09	0.000	6.116654	6.427524
_cons	18.84218	.6745715	27.93	0.000	17.52004	20.16431

xtcorr

Estimated within-Id correlation matrix R:

	c1	c2	c3	c4	c5	c6	c7	c8	c9
r1	1.0000								
r2	0.9167	1.0000							
r3	0.8403	0.9167	1.0000						
r4	0.7702	0.8403	0.9167	1.0000					
r5	0.7061	0.7702	0.8403	0.9167	1.0000				
r6	0.6472	0.7061	0.7702	0.8403	0.9167	1.0000			
r7	0.5933	0.6472	0.7061	0.7702	0.8403	0.9167	1.0000		
r8	0.5438	0.5933	0.6472	0.7061	0.7702	0.8403	0.9167	1.0000	
r9	0.4985	0.5438	0.5933	0.6472	0.7061	0.7702	0.8403	0.9167	1.0000

$$\text{Cov}(Y_{ij}, Y_{ik}) = \sigma^2 \rho^{|j-k|}$$